A Lower Limit on the Age of the Universe

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Abstract

We report the results of a detailed numerical study designed to estimate both the absolute age and the uncertainty in age (with confidence limits) of the oldest globular clusters. Such an estimate is essential if a comparison with the Hubble age of the universe is to be made to determine the consistency, or lack thereof, of various cosmological models. Utilizing estimates of the uncertainty range (and distribution) in the input parameters of stellar evolution codes we produced 1000 Monte Carlo realizations of stellar isochrones, with which we could fit the ages of the 18 oldest globular clusters. Incorporating the observational uncertainties in the measured color-magnitude diagrams for these systems and the predicted isochrones, we derived a probability distribution for the mean age of these systems. The one-sided 95% C.L. lower bound for this distribution occurs at an age of 12.07 Gyr. This puts interesting constraints on cosmology which we discuss. Further details, including a description of the distributions, covariance matrices, dependence upon individual input parameters, etc. will appear in a future article.

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The apparent dichotomy between the upper bound on the age of the Universe obtained from the Hubble Constant and the lower bound obtained by dating the oldest globular clusters in our galactic halo represents one of the most significant potential conflicts in modern observational cosmology. For Hubble constant $H_0 = 100\ h\ \mathrm{km\,sec^{-1}Mpc^{-1}}$, a flat, matter dominated universe has an age given by

$$\tau_{Hubble} = 2/3\ H_0^{-1} = 6.6h^{-1}Gyr \tag{1}$$

If the universe is open, the factor of 2/3 is changed to unity, but even in this case if the age of the oldest globular clusters in our galaxy is really 16 Gyr, as various best fits estimates recently suggest [1, 2], then this will be inconsistent with the Hubble age if $h > .6$. Consistency with the flat matter dominated universe estimate is impossible if $h > .4$. Since several recent estimates of the Hubble constant based on Type 1a supernova, and Hubble Space Telescope observations of Cepheid variables in the Virgo cluster both tend to suggest $h > .65$ [3, 4], this has been one factor which has led various groups to argue once again for the need for a cosmological constant (i.e. [5]).

Since the Hubble estimate is unambiguous for a fixed Hubble constant the crucial uncertainty in this comparison resides in the globular cluster (GC) ages estimates themselves. Rough arguments have been made that changes in various input parameters in the stellar evolution codes designed to derive globular cluster isochrones, or in the RR Lyrae distance estimator used to determine absolute magnitudes for GC stars, might change age estimates by 10-20 % (i.e. [6]). However, no systematic study has yet been undertaken to realistically estimate the cumulative effect of all existing observational and theoretical uncertainties in the GC age analysis. This is the purpose of the present work.

One of the reasons we believe such an analysis had not yet been carried out is that it is numerically intensive. Each run of a stellar evolution code for a single mass point takes 3-5 minutes on the fastest commercially available workstations. Nine different mass points at three different metallicity values must be run to produce each set of isochrones. If one then runs, say, 1000 different isochrone sets to explore the different parameter ranges available, this requires over 8 weeks of continuous processing time.

This is long, but not prohibitive, so because of the importance of this issue, we developed the Monte Carlo algorithms necessary for the task. This involved first examining the measurements of input parameters in the stellar evolution code to determine their best fit values, and also their uncertainties along with the appropriate distributions to use in the Monte Carlo. Then the stellar evolution code [7] and isochrone generation code were rewritten to allow sequential input of parameters chosen from these distributions, and output of the necessary color-magnitude (CM) diagram observables. Finally, we derived a fitting program to compare the predictions to the data. Since the numerically intensive part of this procedure involves the Monte Carlo generation of isochrones, by incorporating the chief observational CM uncertainty afterwards our results can quickly be refined as this uncertainty is refined.

In this article we briefly describe the general features of our analysis and present our main result, the derived probability distribution for the age of the oldest galactic globular clusters. This should be the result which is of broadest and most immediate general interest. In a future work we shall describe the details of the analysis, and present our results for covariance matrices, correlation functions, dependence on individual parameters, and observational uncertainties. These results will be of more use to researchers

in the field of stellar evolution and globular cluster dating.

Monte Carlo Analysis Inputs: General Features

We have focussed on what we believe are the chief input uncertainties in the derivation of stellar evolution isochrones. These include: pp and CNO chain nuclear reaction rates, stellar opacity uncertainties, uncertainties in the treatment of convection and diffusion, helium abundance uncertainties, and uncertainties in the abundance of the α-capture elements (O, Mg, Si, S, and Ca). Our stellar evolution code was revised to allow batch running with sequential input of these parameters chosen from underlying probability distributions. We did not include the equation of state among our Monte Carlo variables as it is now well understood in metal-poor main sequence stars. It has recently been shown that the detailed equation of state by Rogers [8] gives very similar globular cluster age estimates to those obtained using the Debye-Hückel correction [9], which was used in this study.

We describe briefly our parameter choices and distributions below: We included uncertainties for the 3 most important reactions in the pp chain. For $p + p \rightarrow {}^2H + e^+ + \nu$ we used the analysis of Kamionkowski and Bahcall ([10]), except that where they took theoretical errors as gaussianly distributed, we decided that a uniform distribution better represented the state of our (lack of) knowledge. There are two sources of theoretical error, one from the uncertainty in the particle wave-functions, another from meson exchange ([11]). We use a relative modification to this reaction of $1 \pm .002 \; {}^{+.0014}_{-.0009} \; {}^{+.02}_{.012}$, where the 2nd term is 1-sigma gaussian, and the 3rd and 4th are top hat distributions. For 2 other *pp* chain reactions, ${}^3He + {}^3He \rightarrow {}^4He + p + p$, and ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$, the uncertainties included in this study were taken from Bahcall and Pinsonneault ([12], Table I). The CNO nuclear reaction

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rates and their uncertainties are from Bahcall ([13]; Table 3.4).

The stellar opacities are broken into high temperature and low temperature regimes. For the high temperature regime ($T > 10^4$ K), the OPAL opacities [14] are utilized. The uncertainties in these opacities were evaluated by a comparison to the LAOL opacities [15]. In the temperature regime relevant for nuclear fusion ($T \gtrsim 6 \times 10^6$ K), typically differences of 1% were found, with maximum differences of 3%. Thus, we elected to multiply the high temperatures opacities by a Gaussian distribution, with a mean of 1, and $\sigma = 0.01$. The Kurucz opacities [16] were used for the low-temperature regime. Low temperature opacities calculated by different groups can differ by a large amount [17], though modern calculations appear to agree to within ~ 30%. In addition, we intercompared Kurucz's calculations for different element mixtures, finding maximum differences of 30%. For these reasons, we elected to multiply the Kurucz opacities by a number which was uniformly drawn from the range $0.7 - 1.3$.

The correct treatment of the convective regions of the star has been a long standing problem in stellar astrophysics, which stems from our poor understanding of highly compressible convection and its interaction with radiation in the optically thin outer layers. Most stellar evolution codes use the mixing length approximation, which is a simple model of convection where blobs of matter are supposed to rise or fall adiabatically over some distance, and then instantaneously release their heat [18]. The uncertainties in this treatment of convection are parameterized into a single non-dimensional quantity called the mixing length, which is usually taken to be fixed during a star's evolution. Its value is typically chosen by requiring that a solar model have the correct radius and luminosity at the solar age, or by a comparison of theoretical isochrones to observed CM diagrams. A mixing length of around

1.8 appears to provide a reasonable match to the observations, but its exact value depends on the input physics (opacities, model atmospheres, etd.) used to construct the stellar models. Modern solar models typically employ mixing lengths between 1.7 and 2.1 [19, 20]. To further explore this issue, we have conducted tests whereby isochrones were constructed from models with different mixing lengths and compared to a metal-poor globular cluster CM diagram. We found that changes in the mixing length of 0.3 could be ruled out as the isochrones no longer fit the data, if all other input parameters were held constant. Allowing for possible variation among globular clusters and for the fact that a broader range in mixing length might fit the data if other parameters are allowed to vary at the same time, we elected to use a rather broad Gaussian distribution with a mean of 1.85 and $\sigma = 0.25$.

Whether or not to include the effects of element diffusion (whereby helium tends to sink to the center of the star, while hydrogen is rises to the surface) is a difficult question. Physical models of compressible plasmas suggest that diffusion should be occurring in stars, and predict diffusion coefficients with a claimed accuracy of about 30% [21]. However, these models assume that no other mixing process occurs within the radiative regions of a star, which may not be true. Helioseismology appears to suggest that diffusion is occurring in the sun [22], but the evidence is not compelling [19]. Models of halo stars which incorporate diffusion predict curvature in the Li-effective temperature plane which is not observed [23], which suggests that some process is inhibiting diffusion in halo stars. Given these uncertainties, we have elected to multiply the diffusion coefficients given by Michaud & Proffitt [21] by a number uniformly drawn from the interval $0.3 - 1.2$. The use of a flat distribution with a rather large range reflects our conviction that the incorporation of diffusion within stellar models is subject to large uncertainties.

The primordial helium abundance, relevant to old halo stars, and an important input in our stellar codes, has taken on renewed interest as a result of recent calculations of Big Bang Nucleosynthesis (BBN) light element production [24, 25, 26]. In order to BBN estimates to agree with inferred primordial abundances, significant systematic uncertainties must be allowed for. It has become clear that such uncertainties are the dominant feature of the comparison between theory and observation. We therefore utilize here a flat distribution for the primordial helium mass fraction between 0.22 and 0.25, which encompasses the range of recent estimates.

In order to calculate a stellar model, the abundance of the elements heavier than helium (denoted by Z) must be specified. Due to its numerous spectral lines, it is relatively easy to determine the abundance of Fe in globular cluster stars. Unfortunately, the abundances of the other heavy elements are more difficult to determine and it has been common to assume that the other heavy elements are present in the same proportion as they are in the Sun. However, from both theoretical arguments and observational evidence, it is clear that the elements which are produced via α-capture are enhanced in abundance relative to their solar value. It is relatively easy to incorporate the effects of the enhancement of the α-elements on the stellar models by redefining the relationship between the model Z and the iron abundance [27]. Oxygen is by far the most important of the α-capture elements, as it acounts for roughly half of all the heavy elements (by number) present in the Sun. For this reason, we concentrate on observations of oxygen as representative of the α-capture elements. The determination of oxygen abundances in stars is extremely difficult, and subject to a number of systematic uncertainties (e.g. [28]). Recently, high quality [O/Fe][2] abundances for a number of halo

[2]We use the common spectroscopic notation, where the abundance of element y relative

stars have been obtained [29], with the result that the mean abundance was found to be [O/Fe] = 0.55 ± 0.05, where the error is simply the standard deviations of the measurements. In addition to this error, one must add in the possible systematic errors which have been discussed by a number of authors [28, 30, 31]. An analysis of the literature has lead us to conclude that possible systematic errors in the determination of [O/Fe] may be as large as ± 0.2 dex. Thus, the abundance of the α-elements was taken to be $[\alpha/\text{Fe}] = 0.55 \pm 0.05 \pm (\text{Gaussian}) \pm 0.2 \,(\text{top-hat})$.

Finally, a color table must be used to convert our theoretical luminosities and temperatures to observed magnitudes and colors. The construction of an accurate color table requires the use of theoretical model atmospheres, which are still subject to large uncertainties. We elected to take this uncertainty into account by randomly choosing one of two totally independent color tables [32, 33] with equal probability in constructing each isochrone set in the Monte Carlo. These two tables span reasonably the present range used to transform from theoretical temperatures and luminosities to observed colors and magnitudes.

The Fitting Procedure and the Probability Distribution for Globular Cluster Ages

Once a set of isochrones (which consists of isochrones of three different metallicities for a set of 15 different ages between 8 and 22 Gyr) is derived, the comparison with the observed parameters of a specific set of globular clusters requires a fitting procedure. In order to minimize the large uncertainties in the effective temperatures of the models [34], we have elected to use the difference in magnitude between the main sequence turn-off, and the horizontal

to element x is denote by $[y/x] \equiv \log(y/x)_{\text{star}} - \log(y/x)_{\text{sun}}$.

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branch (HB, in the RR Lyr instability strip) as our age diagnostic. This age determination technique is commonly referred to as ΔV_{HB}^{TO} and has been extensively used in the astronomical literature (e.g. [35]). Our isochrone sets provide us with the main-sequence turn-off luminosity as a function of age and metallicity. Due to the importance of convection in the nuclear burning regions of HB stars, theoretical HB luminosities are subject to large uncertainties, and so we have elected to combine our theoretical main-sequence turn-off luminosities with an observed relation for the luminosity of the HB (see below). This results in a grid of predicted ΔV_{HB}^{TO}'s as a function of age and [Fe/H], which is then fit to an equation of the form

$$t_9 = \beta_0 + \beta_1 \Delta V + \beta_2 \Delta V^2 + \beta_3 [Fe/H] + \beta_4 [Fe/H]^2 + \beta_5 \Delta V [Fe/H], \qquad (2)$$

where t_9 is the age in Gyr. The observed values of ΔV_{HB}^{TO} and [Fe/H], along with their corresponding errors, are input in (2) to determine the age and its error for each GC in our sample.

There is abundant evidence for a large age range within different GC systems (e.g. [36, 37]) so one must take care to select a sample which only includes old globular clusters. Observational errors in the determination of the turn-off and horizontal branch magnitudes lead to a $\sim 10 - 20\%$ error in the derived age of any single cluster. Thus, to minimize the observational uncertainties, it is best to determine the mean age of a number of GCs. In light of the strong evidence for an age-metallicity relationship (with metal-poor clusters being the oldest), only metal-poor clusters were selected ([Fe/H] ≤ -1.6). From this list of metal-poor clusters, any cluster which has been shown to be young using the difference in color between the giant-branch and main sequence turn-off [37], or which is suspected of being young due to its unusually red horizontal branch for its metallicity [38] was dis-

carded. From the sample of 43 GCs [36] for which high quality observations are available, 27 survived the metallicity cut, of which 10 were discarded for being young, leaving a total of 17 globular clusters. Our final sample contained the following clusters: NGC 1904, 2298, 5024, 5053, 5466, 5897, 6101, 6205, 6254, 6341, 6397, 6535, 6809, 7078, 7099, 7492, and Terzan 8.

One of the things we checked is the the inferred dispersion in the age of the 17 globular clusters is not larger than that which we expect based on the observational uncertainties. Using the uncertainties in the individual ages determined from uncertainties in the observed turnoff magnitude and metallicity, we examined the dispersion about the mean age for the 17 clusters using a χ^2 test. We found a reduced χ^2 of 0.55 per degree of freedom, indicating both no evidence for any intrinsic dispersion in age for our sample, and that the quoted observational uncertainties for each cluster may be too generous. In any case, given the quoted accuracy, it is certainly consistent to assign a single mean age for the sample.

One chief observational uncertainty common to all globular clusters is retained until the end of the analysis. This is the M_v determination for RR-Lyrae variables. In order to determine ΔV_{HB}^{TO} as a function of age and metallicity, we combine our theoretical turn-off magnitude with an observationally based estimate for the absolute magnitude of the HB, in the RR Lyr instability strip (hereafter referred to as $M_v(RR)$). There are a number of independent, observationally based techniques which can be used to derive $M_v(RR)$. In general, it has been found that the absolute magnitude of the RR Lyr stars can be represented by an equation of the form

$$M_v(RR) = \mu \, [Fe/H] + \gamma, \tag{3}$$

where μ is the slope with metallicity and γ is the zero-point. Note that

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$M_v(RR)$ is independent of age (at least, with systems greater than 8 Gyr old). Recent estimates for the slope with metallicity vary from 0.15 to 0.30 [35, 36]. Fortunately, since we are determining the mean age of 17 globular clusters in the restricted metallicity range $-2.41 \leq [Fe/H] \leq -1.60$, the uncertainties in the slope has only a small effect on our age estimate. Tests which we conducted indicated that the maximum difference in our mean age was only 0.5% when the slope was varied between 0.15 and 0.30. The most recent work suggests that $\mu = 0.20$ is likely to be correct [39, 40] which is the value we have used in this study. Uncertainties in the zero-point, γ in eq. 3 have a large impact on our derived age estimates. For this reason, we have spent considerable time reviewing recent observational estimates of the zero-point. These estimates for the zero-point are usually given as a $M_v(RR)$ value at a specific metallicity. As the globular clusters in our sample have a median metallicity of $[Fe/H] = -1.82$ and a mean metallicity of $[Fe/H] = -1.93$ we have elected to transform the various zero-point estimates to $[Fe/H] = -1.90$ using a slope of $\mu = 0.20 \pm 0.04$.

Layden Hanson & Hawley [41] have recently used the statistical parallax technique to determine $M_v(RR) = 0.68 \pm 0.12$ in field halo RR Lyr stars. Walker [42] measured the apparent magnitude of LMC RR Lyr stars, and assumed an LMC distance modulus of 18.5 to infer $M_v(RR) = 0.44 \pm 0.10$. This choice for the distance modulus of the LMC was based on the Cepheid distance, main sequence fitting and the SN1987A ring distance to the LMC. This last method is a purely geometrical method, and should be the most reliable. However, the SN1987A distance to the LMC has recently been revised to 18.37 [43], implying $M_v(RR) = 0.57 \pm 0.10$. Main sequence fitting of globular cluster CM diagrams to local halo stars with well determined parallaxes can be used to determine the distance to globular clusters, and

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hence, $M_v(RR)$. Unfortunately there is only one relatively metal-rich sub-dwarf which has a well determined parallax. Application of this technique to the globular cluster M5 yields $M_v(RR) = 0.76 \pm 0.12$ [35]. The only direct determination of $M_v(RR)$ in a metal-poor globular cluster is by Storm, Carney & Latham [44]. They used a Baade-Wesselink/infrared flux analysis to determine $M_v(RR) = 0.52 \pm 0.26$. Although the error is large due to possible systematic uncertainties, it does suggest that the RR Lyr stars in metal-poor globular clusters are somewhat brighter than those found in the field [41], or in metal-rich globular clusters [35]. In light of the above estimates, we have elected to use $M_v(RR) = 0.60 \pm 0.08$ (corresponding to $\gamma = 0.98 \pm 0.08$). This central value was chosen by a straight average of the 4 published $M_v(RR)$ estimates referenced above. The error bar was chosen to ensure that the $1\,\sigma$ range would include the central value obtained for $M_v(RR)$ in a metal-poor globular cluster, and the $2\,\sigma$ range ($0.44 - 0.76$) would encompass all of the estimates quoted above. Our choice is further supported by a study which just appeared comparing four different distance estimators, including kinematic, RR Lyr, Cepheid and Type II Supernovae for consistency [45], and found that this appeared to require a range for $M_v(RR)$ similar to the one we chose.

Figure 1 displays the ensemble of age estimates from our Monte Carlo, for different values of $M_v(RR)$. It was generated as follows: For each of the sets of isochrones and given a value for $M_v(RR)$ we determine a mean age and 1σ uncertainty in the mean. In the figure we show these values for each isochrone set, for 3 values of $M_v(RR)$: the mean, .60, and the endpoints of our 95% C.L. range, .44 and .76. In order to obtain the final histograms displayed as figs 2 a-b, we follow an analogous procedure, but this time allow $M_v(RR)$ to be a random variable, and sample the sets of isochrones with

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replacement 12,000 times. For each sample, rather than the mean age, we record a random age drawn from a gaussian distribution with the mean age and variance for that isochrone set at that $M_v(RR)$. Then the data are sorted and binned to produce the figures: 2(a) the full distribution for the assumed Gaussian spread in $M_v(RR)$ of $\pm.08$, and 2(b) the distribution for fixed value of $M_v(RR)$ of 0.6. In this way the effect of the uncertainty in $M_v(RR)$ can be explicitly examined.

Conclusions

Our results indicate that at the one sided 95% confidence level (determined by requiring 95% of the determined ages to fall above this value) a lower limit of approximately 12.1 Gyr can be placed on the mean value of these 18 Globular clusters. (The symmetric 95% range of ages about the mean value of 14.56 Gyr is 11.6-18.1 Gyr.) Note that the distribution deviates somewhat from Gaussian, as one might expect. In particular, at the lower age limits the rise is steeper than Gaussian, reflecting the fact that essentially all models give an age in excess of 10 Gyr, while the tail for larger ages is larger than gaussian. The explicit effect of the largest single common observational uncertainty, that in $M_v(RR)$, increases the net width of the distribution by approximately ±0.6 Gyr (i.e. $\approx \pm5\%$). Note that simply varying $M_v(RR)$ over its full 2σ range, keeping all other parameters fixed, would produce a $\pm16\%$ change in GC ages estimates. For comparison, the next most significant input parameter uncertainties in this same sense are $[\alpha/Fe]$ ($\pm7\%$ effect), mixing length ($\pm5\%$ effect), and diffusion, [14]Np reaction rate, and primordial Helium abundance, each of which would affect age estimates at the $\pm3\%$ level if allowed to vary over its entire range, keeping all other parameters fixed.

We believe that this result can now be used with some confidence to compare to cosmological age estimates. Of course, in addition to the age determined here one must add some estimate for the time it took our galactic stellar halo to form from the initial density perturbations present during the Big Bang expansion. Estimates for this formation time vary from 0.1 – 2 Gyr. To be conservative, we choose the lower value. In this case, we find that the age of globular clusters in our galaxy is inconsistent with a flat, matter dominated universe unless $h < 0.54$, and for a nearly empty, matter dominated universe unless $h < 0.80$. If the value of h is definitely determined to be larger than either of these values, some modification, such as the addition of a cosmological constant, would seem to be required.

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Figure Captions

Figure 1: The suite of Models generated by the Monte Carlo Procedure. For each Model the $1 - \sigma$ age range is plotted for 3 choices of $M_v(RR)$. The red, green and blue data points show the age variation for $M_v(RR) = 0.44$, 0.60 and 0.76 respectively.

Figure 2: The histograms exhibit the relative numbers of realizations of mean globular cluster ages drawn randomly from the Monte Carlo data set (with uncertainties on individual age estimates taken to be Gaussian) using a value for $M_v(RR)$ (the absolute magnitude of the RR Lyrae Variables) chosen from one of two different distributions: a) a Gaussian, $M_v(RR) = 0.6 \pm .08$, b) a Delta function, $M_v(RR) = 0.6$. The dashed line is a Gaussian approximation to the actual distribution. The mean and standard deviation of the Gaussians are shown in the Legend. The horizontal arrows show the 1 and 2-σ ranges in age based upon the Gaussian approximation. The one-sided 95% Confidence Limit for a lower bound on the age of the Universe is displayed as an arrow extending to the right from a vertical bar. This is calculated directly from the generated distribution.
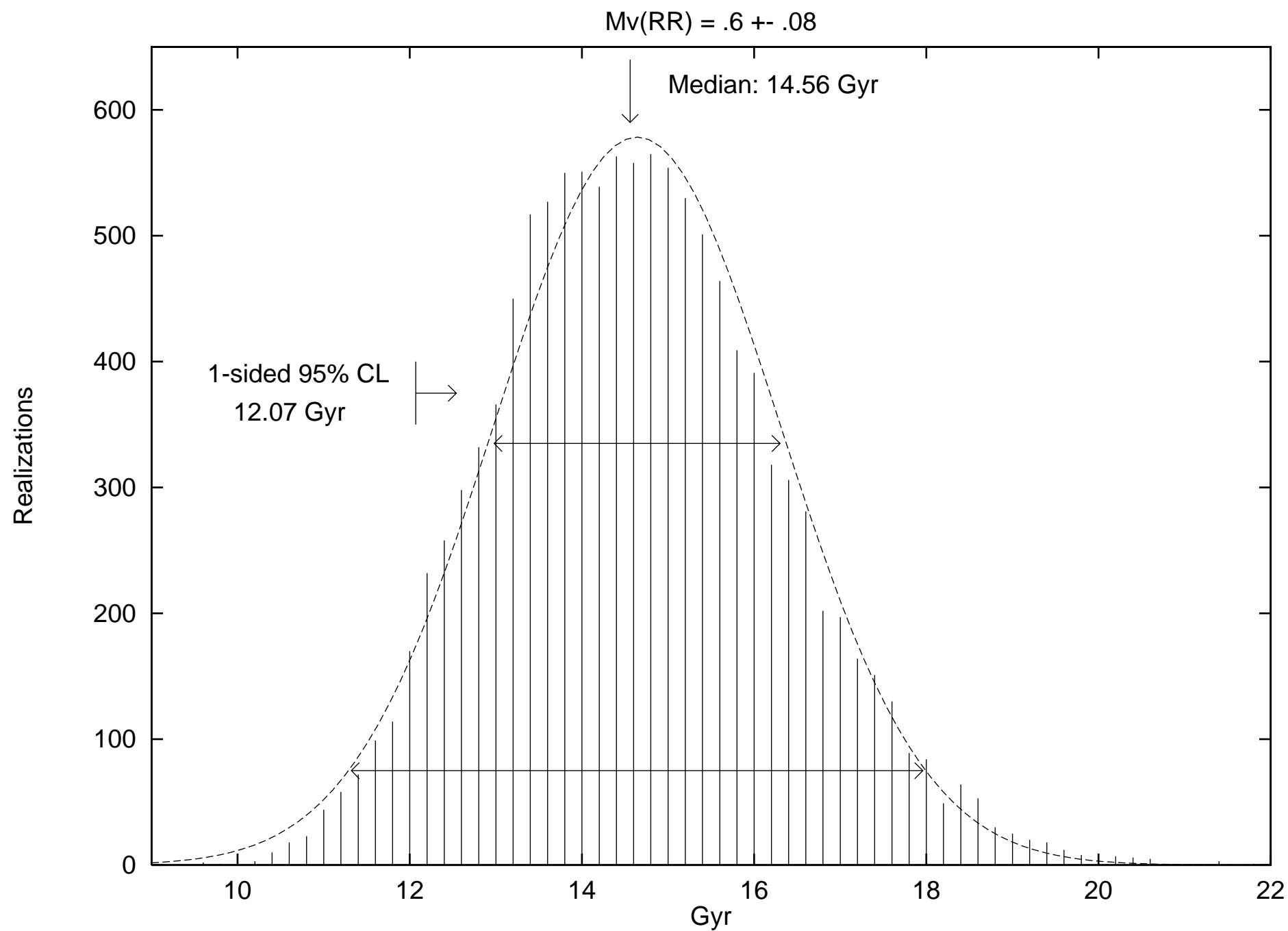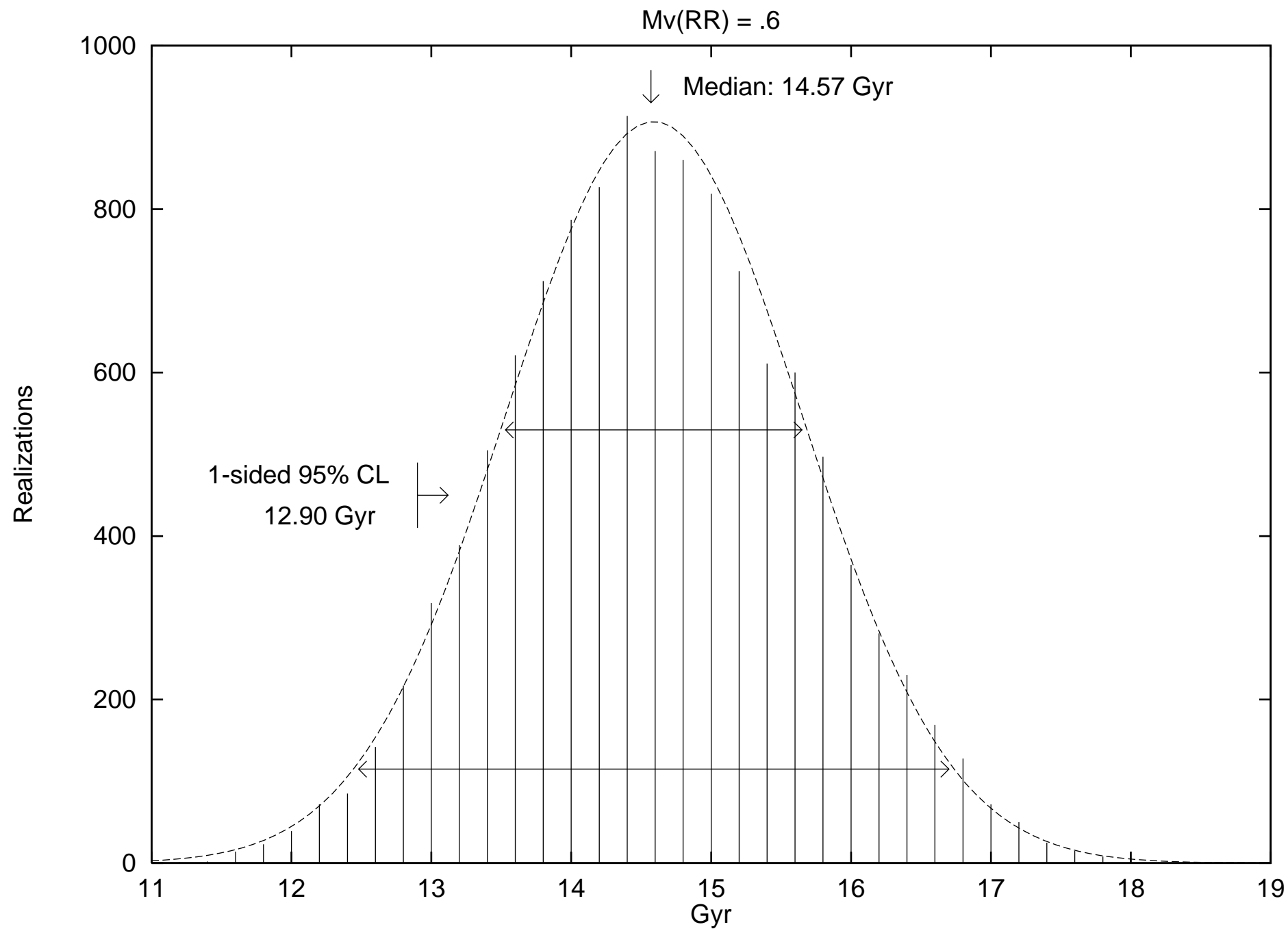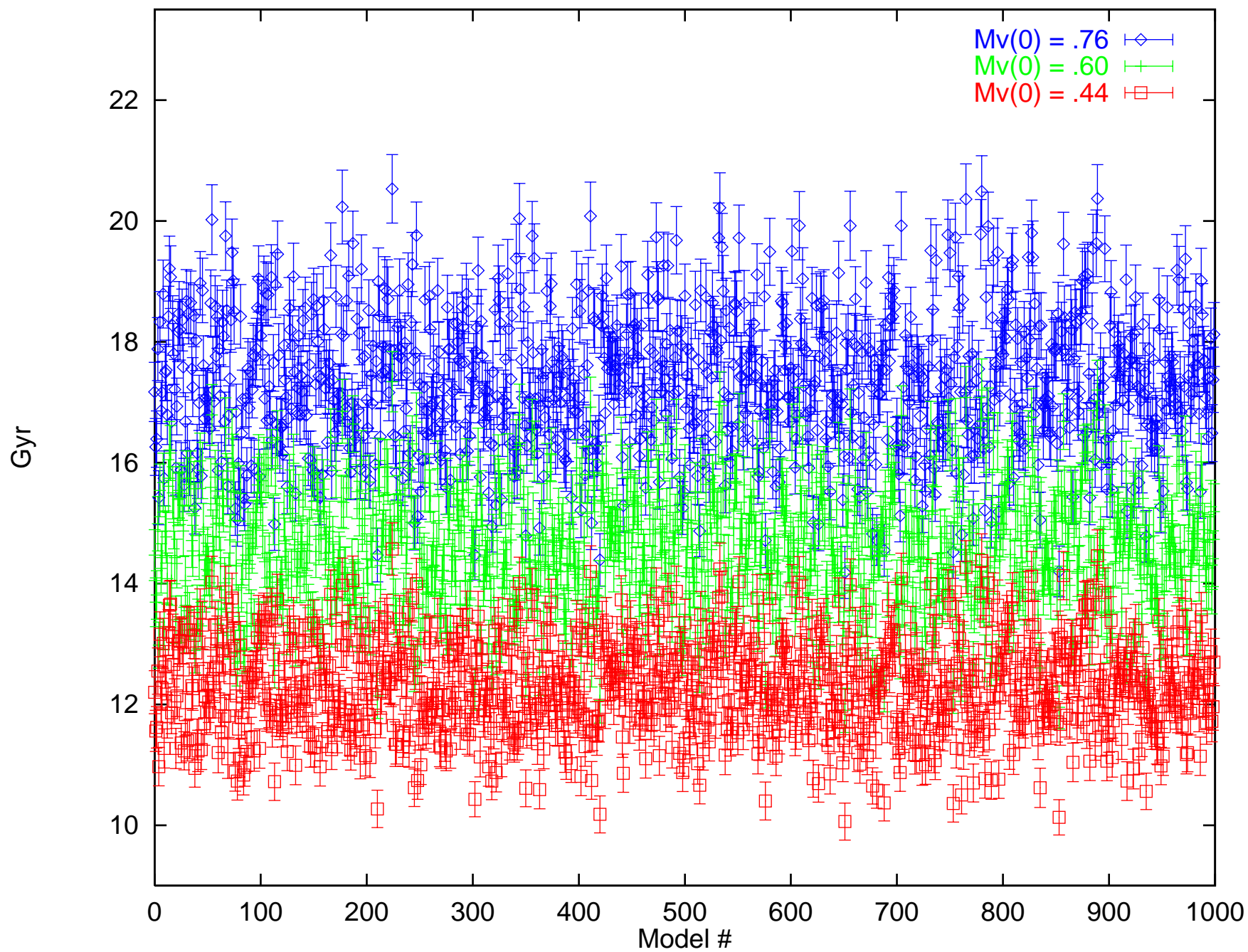
Figure 2a

Figure 2b

Figure 1